**Time-dependent risk measurement for stock index arbitrage**[1]

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## Abstract

This paper conducts an empirical analysis of the mispricing of stock index futures. We measure the uncertainty surrounding the magnitude of the dividend yield and interest rate to maturity of the futures contract as well as the timing of announcements that resolve these uncertainties inherent in pricing index futures. We allow for the value that investors place on the cash dividends and franking credits distributed by the underlying stocks when estimating mispricing levels, to overcome a deficiency in previous mispricing studies that ignore the plausible values placed on these dividend components by investors in the contemporary Australian dividend imputation context. We also make use of more precise estimates of transaction costs than used in previous mispricing studies. We find that the timing of dividend announcements and the volatility of the index have the most significant effects in widening the arbitrage window for index futures.

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## 1. Introduction

The linkage in prices between index futures and their underlying stocks is maintained by arbitrageurs. In the absence of basis risk, transaction costs and taxes, arbitrageurs quickly close the gap between the price of the index futures and its substitute portfolio of constituent stocks whenever a profit opportunity arises. Transaction costs imply that the futures price can fluctuate within bounds around the theoretical value of the contract without triggering a profitable opportunity. The introduction of risk into the arbitrage process, such as occurs when dividend yields and interest rates to maturity of the contract are uncertain, means that either the arbitrage window is further widened and/or these additional risks need to be priced into the 'fair' value of the contract. The extent of deviations from theoretical value impacts upon the effectiveness of index futures as a tool for hedging market risk and for taking active positions to gain exposure to the underlying equities.

In Section 1 we outline the determinants of index futures mispricing considered in previous research and the potential to improve the way in which these variables are measured and modelled. Section 2 provides the institutional setting, data and specifies our measurements. We report our results in section 3 and conclude in section 4.

### 1.1 Determinants of index futures mispricing

Greater absolute magnitudes of mispricing for longer times to maturity which have been observed in stock index futures markets are consistent with arbitrage being more risky further out from maturity (MacKinlay and Ramaswamy, 1988; Yadav and Pope, 1994). Arbitrageurs require greater compensation to act upon deviations from theoretical pricing levels when the risks they face are higher, permitting larger deviations to be sustained early in the futures expiry cycle. MacKinlay and Ramaswamy (1988) identify three of the risks that are greater with longer times until expiration: (i) the risk of unanticipated increases or decreases in dividends; (ii) unanticipated interest earnings or costs from financing the marking-to-market flows from futures positions; and (iii) attempts at arbitrage motivated trading that employ less than the full basket of stocks in the index must allow for a greater margin of error with longer times to expiration. We endeavour to disentangle which of the risks inherent in index arbitrage activity play the most significant role in obstructing the extent to which arbitrageurs can drive prices towards theoretical levels further out from maturity. In particular, we focus on the uncertainty surrounding two key factors that feature in the cost of carry model for stock index futures, the dividend yield and interest rate to maturity.

Existing evidence linking the trend in mispricing across the index futures expiry cycle to the resolution of dividend yield uncertainty is weak. Analyst forecasts for the sizes of dividends spanning the period to futures expiry are less reliable further out, dependent upon early guidance from a greater number of firms. Special dividends can constitute a substantial fraction of total dividends and are difficult to predict. The likelihood of incorrectly predicting ex-dividend dates is also higher for longer times until maturity. The risk pertaining to the unknown timing of ex-dividend dates is especially relevant to the pricing of the near futures contract, in cases when either dividends are delayed that were expected to have ex-dividend dates before the expiration of the contract or dividends are brought forward that were expected to be deferred until after the expiration of the contract. Despite the presence of these several forms of dividend uncertainty, Yadav and Pope (1994) are unable to attribute the absolute magnitude of mispricing they observe in FTSE 100 Index Futures to dividend forecast errors. Their measure of the uncertainty surrounding the timing of future dividends focuses on the difference between the ex-dividend date and the actual dividend payment date, which does not capture the pricing consequences of firms rescheduling ex-dividend dates relative to previous years. To address this issue more effectively, we construct alternative measures based on the timing of dividend announcements heralding ex-dates both pre and post futures expiry.

With the financing cost determined at the time of entering into a futures contract, index arbitrageurs are exposed to interest rate risk if the borrowing or lending they undertake to support the cash leg of their transactions does not match the expiry dates of the futures. The cost of continually rebalancing the maturities of cash and futures positions to neutralise this interest rate exposure may be prohibitive. Unanticipated changes in interest rates spanning the period to futures expiry are more likely to occur when starting further out. The Reserve Bank Board formulates monetary policy with regard to developments in the Australian and international economies. Financial markets reassess the likely outcome of subsequent central bank decisions on interest rates by analysing macroeconomic information as it is released. If interest rate risk is a relevant factor for index futures pricing, absolute mispricing should be higher when the ex-ante volatility of the interest rate over the period to contract maturity is higher. In this paper, we investigate the dependence of absolute mispricing on the ex-ante estimate of volatility implied in interest rate option prices. Then we supplement our analysis of interest rate uncertainty by constructing an alternative measure that describes the frequency of the economic releases that influence interest rate expectations over the period to futures expiry.

MacKinlay and Ramaswamy (1988) provide evidence of countervailing forces that serve to establish a narrower trading band for index futures prices. Arbitrageurs' option to unwind their positions prematurely introduces path dependence into the mispricing series. In particular, MacKinlay and Ramaswamy show that conditional on the mispricing of S&P 500 futures contracts having crossed one arbitrage bound it is less likely to cross the opposite bound. This phenomenon is consistent with the notion that arbitrageurs unwind positions established when the mispricing was outside one bound before it reaches the other bound. It is optimal to close out these positions before putting on new arbitrage trades in the reverse direction[2]. The early unwind option potentially mitigates against the greater risk inherent in arbitrage activity further out from maturity. Arbitrageurs also obtain the option to roll their futures positions forward into the next available maturity.

Greater absolute magnitudes of mispricing for longer times to maturity are also consistent with higher implicit transaction costs producing a wider arbitrage window further out from maturity, even before the risks faced by arbitrageurs are taken into account. The presence of transaction costs implies that the price of the index futures

[2] If the mispricing never 'corrects' itself over the life of the contract, arbitrageurs will be obliged to unwind their positions at the opening call auction on the expiry date. Nonetheless, MacKinlay and Ramaswamy (1988) and Yadav and Pope (1994) demonstrate that arbitrageurs have often had the opportunity to unwind their positions at a profit prior to the expiry date, making it difficult to predict the direction of market-on-open orders on the last trading day.

could fluctuate within a band around its theoretical value without representing a potential profit opportunity for even the most favourably situated arbitrageurs (MacKinlay and Ramaswamy, 1988). The width of the band is determined by explicit costs such as fees paid to exchanges and transaction taxes and from implicit costs including the bid-ask spreads and the price impact costs of opening up positions in both the stock and futures markets[3]. We employ McInish and Wood's (1992) time weighting procedure to produce more precise estimates of bid-ask spreads in both the stock and futures markets than used in previous mispricing studies. Including these time weighted measures in our modelling allows us to control more effectively for fluctuations in the width of the arbitrage window that occur due to day-to-day variations in the transaction costs associated with laying on index arbitrage trades. In this way, we are able to assess the added impact of the risks faced by arbitrageurs on width of the arbitrage window at different times to maturity.

## 2. Institutional setting and data

Introduced in April 2000, the S&P/ASX 200 index is recognised as the investable benchmark for the Australian equities market. The index was converted from a market capitalisation weighted index to a free float based index on 1 October 2002. The weighting of constituents is determined by the free float assigned to each stock by the Standard and Poor's Australian Index Committee. The stocks comprising the index are traded on the Australian Stock Exchange's (ASX) computerised trading system, known as the Stock Exchange Automated Trading System (SEATS) until October 2006. The level of the S&P/ASX 200 is calculated by Standard & Poor's and is reported to the market every 30 seconds.

The Sydney Futures Exchange's (SFE) main equity index contract, *SFE SPI 200™ Index Futures*, is written over the S&P/ASX 200 index. The SFE's electronic trading system, Sydney Computerised Market (SYCOM®), operates 24 hours a day. The system allows brokers to route client orders from computers located in their offices to the central market. The day session commences at 9:50 a.m. and finishes at 4:30 p.m. for SFE SPI 200™ futures. In contrast, the stocks from which the index is constructed are traded on the ASX in continuous auction mode from 10:00 a.m. to 4.00 p.m.[4]. The contracts we examine expire in March, June, September and December each year. The minimum tick for SFE SPI 200™ futures is one index point representing a fixed amount of AUD 25 per contract. Transaction prices and volumes and recorded and reported immediately. Though six maturities are listed at any given time, we confine our analysis to the nearest expiry contract (denoted the 'near' contract) which has the most substantial trading volume.

For December 2001 and all subsequent quarterly SFE SPI 200™ contracts, the final settlement price (Special Opening Quotation) is calculated using the first traded price of each component stock in the S&P/ASX 200 index on the last trading day being the

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[3] The bid-ask spread and price impact costs of closing out both the stock and futures positions can be avoided by holding the positions until the last trading day for the futures contract and employing market-on-open orders in the respective markets.

[4] Stocks on the ASX do not open simultaneously. Rather, they are grouped according to their alphabetical ranking and each group is opened randomly at different times between 10:00 a.m. and 10:09 a.m..

third Thursday of the month (denoted day 0 in this article). Trading in expiring contracts ceases at 12.00 p.m. on day 0.

### 2.1 Data

Reuters trade and quote data, provided by the Securities Industry Research Centre of Asia-Pacific (SIRCA), consists of time-stamped transaction data for transactions in the SFE SPI 200$^{TM}$ futures contracts from 1 January 2002 to 15 December 2005. This provides a structural break free data set of 16 SFE SPI 200$^{TM}$ futures expiry cycles for analysis. Each record contains (in addition to contract identification, time stamp, price and size in lots), information which tags that record as a trade, a bid or an offer. S&P/ASX 200 stock index quotes, time-stamped approximately 30 seconds apart, were also provided to us by SIRCA. The index is updated continuously using transaction prices of the component 200 stocks.

In computing the mispricing series, we use the futures midpoint price (average of the bid and ask prices) outstanding and most recent index update available at the end of each quarter hour interval. Each contract is followed from the expiry date of the previous contract until its expiration.

End-of-day volume figures for SFE SPI 200$^{TM}$ futures and their underlying stocks and open interest figures for SFE SPI 200$^{TM}$ futures were obtained from Bloomberg. The daily free float-adjusted market capitalisation of each stock in the S&P/ASX 200 index and a daily dividend series were also obtained from Bloomberg. The dividend series contains the total actual cash dividends and gross dividends (including imputation tax credits) paid each ex-dividend day by stocks in the S&P/ASX 200. Daily series for the overnight cash, 30, 90 and 180 day bank accepted bills rates were obtained from the Reserve Bank of Australia. The interest rate for loans maturing at the expiration date of the futures is estimated using linear interpolation between these four reference interest rates.

To measure the dividend yield uncertainty for stocks in the S&P/ASX 200 index, analyst-by-analyst fiscal year 1 dividend forecasts for all covered stocks are extracted from the I/B/E/S Daily Detail Earnings Estimates History database. Each dividend forecast record contains broker and analyst codes, the forecast period end date, the estimated dividend in cents per share (DPS), the date the estimate was entered into the database (estimate date) and the most recent date that the estimate was confirmed as accurate (review date). Data on dividends announcements including the announcement dates, ex-dividend dates and payment dates associated with cash dividends and franking credits per share announced by index constituent stocks are obtained from Bloomberg.

Interest rate option contracts based on 90 Day Bank Accepted Bills Futures are traded on the Sydney Futures Exchange and expire on the first Friday of the delivery month for the underlying futures contract. Up to six maturities corresponding to subsequent quarterly bank bill futures expiry dates and several exercise prices were available at any one time. The implied volatility estimates used in this study are those calculated by the Sydney Futures Exchange based on market activity for the purpose of determining daily settlement prices for nearest-to-expiry put and call options which are closest to being at-the-money. Data for macroeconomic news releases were obtained from Bloomberg's *Economic Calendar*.

### 2.2 Variable measurement

Assuming that all investors can borrow and lend at the risk-free interest rate, zero taxes and transaction costs, zero dividend yield and ignoring marking to market, the theoretical price of a futures contract under the cost of carry model is:

$$f_{t,T}(z) = S_t e^{r(T-t)} \tag{1}$$

where $f_{t,T}(z)$ is the current price of the futures contract expiring at time $T$ with zero dividend yield; $S_t$ is the current stock index level, $r$ is the annualised interest rate over the period from time $t$ to time $T$; and $T - t$ is the time to maturity of the contract. The theoretical price of a stock index futures contract with a cash dividend yield $d_c$ is as follows:

$$f_{t,T}(c) = S_t e^{(r-d_c)(T-t)} \tag{2}$$

where the variables in equation (2) are analogous to those in equation (1) and $d_c$ is the annualised cash dividend yield over the period from time $t$ to time $T$. Equivalently, the theoretical price of a stock index futures contract with a gross dividend yield $d_g$ is as follows:

$$f_{t,T}(g) = S_t e^{(r-d_g)(T-t)} \tag{3}$$

where $d_g$ is the annualised gross dividend yield over the period from time $t$ to time $T$.

The 'fair' value of the cash dividends paid out by the stocks over the remaining life of the near futures contract is given by:

$$Cash_t^n = f_{t,T_1}(z) - f_{t,T_1}(c) \tag{4}$$

where $f_{t,T1}(z)$ is the theoretical price of the futures contract with zero dividend yield and $f_{t,T1}(c)$ is the theoretical price of the futures contract with a cash dividend yield $d_c$. In addition, the 'fair' value of the franking credits paid out by the stocks over the remaining life of the near futures contract is given by:

$$Franking_t^n = f_{t,T_1}(c) - f_{t,T_1}(g) \tag{5}$$

where $f_{t,T1}(g)$ is the theoretical price of the futures contract with a gross dividend yield $d_g$.

Beggs and Skeels (2006) analyse the ex-dividend behaviour of share prices in the Australian market from 1986 to 2004. They estimate the gross drop-off ratios (price change at the ex-dividend date divided by the gross dividend), cash drop-off ratios (ratio of the price change at the ex-dividend date to the cash dividend) and franking credit drop-off ratios (ratio of the price change at the ex-dividend date to the franking credit) and consider how these ratios changed in response to changes in the tax regime. They find that the year 2000 tax regime change that allowed for a tax rebate of unused franking credits permanently increased the value of franking credits to the marginal investor and raised the estimated franking credit drop-off ratio. We adopt the estimated cash drop-off ratio of $\gamma_1 = 0.800$ and estimated franking credit drop-off ratio of $\gamma_2 = 0.572$ estimated by Beggs and Skeels for a four-year period (2001 to 2004) after the year 2000 tax change. Thus, the dividend yield used in this study is composed of 80.0 percent of the future fair value of cash dividends and 57.2 percent of the future fair value of franking credits distributed by stocks over the remaining life of the contract divided by the free float-adjusted market capitalisation of stocks underlying the S&P/ASX 200 at the end of the previous trading day.

We demonstrate in appendix 1 that the cash drop-off ratio and franking credit drop-off ratio estimated by Beggs and Skeels (2006) for the Australian stock market from 2001 to 2004 are broadly consistent with the value of cash dividends and franking credits implied in futures prices over our sample period from 2002 to 2005. Accordingly, the theoretical price of a stock index futures contract assuming cash dividends are valued at 80.0 percent of their fair value and franking credits are valued at 57.2 percent of their fair value is as follows:

$$f_{t,T_1}(n) = f_{t,T_1}(z) - 0.800 \times Cash_t^n - 0.572 \times Franking_t^n \qquad (6)$$

where $f_{t,T1}(n)$ is the theoretical price of the futures contract with a nominal dividend yield $d_n$.

The relative level of mispricing is the difference between the market price of the stock index futures contract ($F^n_t$) and its theoretical price, divided by the index value ($S_t$) :

$$M_t^n = [F_t^n - f_{t,T_1}(n)] / S_t \qquad (7)$$

The average intraday mispricing and average intraday absolute mispricing of the near contract are calculated for each day as follows:

$M_t^n =$ $\sum_{j=1}^{N_t} M_t^n(j)/N_t$ is the average intraday mispricing of the near futures index contract on day $t$;

$|M_t^n| =$ $\sum_{j=1}^{N_t} |M_t^n(j)|/N_t$ is the average intraday absolute mispricing of the near futures index contract on day $t$;

$M_t^n(j) =$ mispricing at the $j$th quarter hour mark during day $t$, for the near futures contract relative to the level of the index; and

$N_t =$ number of observations in day $t$.

We examine a variable $UDividend^n_t$ representing the uncertainty about the magnitude of dividends paid by underlying stocks over the remaining life of the futures contract. Fiscal year 1 DPS estimates for the underlying stocks are multiplied by the time remaining to futures contract maturity on the assumption that dividends are paid out uniformly over the year. All estimates that are current on a particular day (indicated by the estimate date and review date) are used to calculate the standard deviation of DPS forecasts for an individual stock. Two assumptions are made in proceeding to construct a measure of dividend uncertainty for the index as a whole from the standard deviations for individual stocks: (i) the spread of (equally weighted) analysts' forecasts represents the probability distribution for future dividends; and (ii) the DPS forecasts for individual stocks are uncorrelated. On the basis of these assumptions, dividend yield uncertainty for the index over the remaining life of the futures contract is given by the weighted average standard deviation of analysts' forecasts for constituent stocks:

$$UDividend_t^n = \frac{\sqrt{\sum_{i=1}^{200}\left(Shares_{i,t} \times StdDev\left(FDPS_{i,t} \times \left(T_1 - t\right)\right)\right)^2}}{\sum_{i=1}^{200}\left(Shares_{i,t} \times P_{i,t}\right)} \qquad (8)$$

where $FDPS_{i,t}$ are analysts' fiscal year 1 dividend per share forecasts for stock $i$ underlying the futures contract expiring in $T_1 - t$ years, $Shares_{i,t}$ is the number of shares in stock $i$ included in the index calculation and $P_{i,t}$ is the closing price of stock $i$ on day $t$.

We also examine variables $ADividend^n_t$ and $ADividend^s_t$ representing the uncertainty about the timing of dividends paid by underlying stocks with ex-dividend dates before and after the expiration of a futures contract respectively. The announcement of dividends with ex-dividend dates falling before the expiry date of the near futures contract could resolve some of the dividend yield uncertainty pertaining to the near contract, if it was unpredictable whether or not some of these dividends would be assigned ex-dividend dates before the expiration of the near contract merely relying upon the timing of corresponding dividends in previous years. Accordingly, $ADividend^n_t$ is defined as the proportion of total gross dividends paid by underlying stocks with ex-dividend dates falling within the current futures expiry cycle (from the expiry date of the previous contract until the expiry date of the current near contract) that are announced over the remaining life of the near contract:

$$ADividend_t^n = \frac{\sum_{i=1}^{200}\sum_{a=t}^{T_1}\left(Shares_{i,t} \times DPS_{i,a}^n\right)}{\sum_{i=1}^{200}\sum_{w=T_0+1}^{T_1}\left(Shares_{i,t} \times DPS_{i,w}^n\right)} \qquad (9)$$

where $DPS^n_{i,a}$ is the gross dividend announced for stock $i$ underlying the futures contract on day $a$ with the relevant ex-dividend date scheduled to occur before the near contract expires at time $T_1$, $DPS^n_{i,w}$ is the gross dividend for stock $i$ with an ex-dividend date $w$ falling between the expiration of the previous futures contract at $T_0$ and the expiration of the current near futures contract at $T_1$ and $Shares_{i,t}$ is the number of shares in stock $i$ included in the index calculation on day $t$.

Furthermore, the announcement of dividends with ex-dividend dates deferred until beyond the expiry date of the near futures contract (tiered ex-dates) could also resolve some of the dividend yield uncertainty pertaining to the near contract, if some of those dividends could have been assigned ex-dividend dates before the expiration of the near contract. $ADividend^{s}_{t}$ is defined as the proportion of total gross dividends paid by underlying stocks with ex-dividend dates in the next futures expiry cycle (from the expiry date of the current near contract until the expiry date of the current deferred contract) that are announced over the remaining life of the near contract:

$$ADividend_t^s = \frac{\sum_{i=1}^{200}\sum_{a=t}^{T_2}\left(Shares_{i,t} \times DPS_{i,a}^s\right)}{\sum_{i=1}^{200}\sum_{w=T_1+1}^{T_2}\left(Shares_{i,t} \times DPS_{i,w}^s\right)} \tag{10}$$

where $DPS^{s}_{i,a}$ is the gross dividend announced for stock $i$ underlying the futures contract on day $a$ with the relevant ex-dividend date scheduled to occur after the current near contract expires at time $T_1$ and before the current deferred contract expires at time $T_2$, $DPS^{s}_{i,w}$ is the gross dividend for stock $i$ with an ex-dividend date $w$ between the expiration of the current near futures contract at $T_1$ and the expiration of the current deferred futures contract at $T_2$ and $Shares_{i,t}$ is the number of shares in stock $i$ included in the index calculation on day $t$.

Assuming that the daily returns on the underlying interest rate security are independent and identically distributed, the volatility implied in interest rate option prices is scaled back to the remaining lifetime of the near futures contract as follows:

$$IVInterest_t^n = AIVIR_t \times \sqrt{T_1 - t} \tag{11}$$

where $AIVIR_t$ is the annualised percentage implied volatility of interest rate options which are closest to being at the money on day $t$ and $T_1 - t$ is the time to maturity of the near futures contract.

Connolly and Kohler (2004) analyse changes in daily implied interest rates from 90 day interest rate futures to identify the Australian macroeconomic news releases which significantly influence interest rate expectations: the consumer price index, employment, the unemployment rate, gross domestic product, building approvals, the trade balance, inventories, investment and retail sales[5]. Hence, we also examine a variable $REconomic^{n}_{t}$ representing the uncertainty related to the timing of these types of economic releases up until the expiration of a futures contract. $REconomic^{n}_{t}$ is defined as the proportion of economic releases falling within the current futures expiry cycle (from the expiry date of the previous contract until the expiry date of the current near contract) that are scheduled to occur over the remaining life of the near contract:

[5] Although we confine ourselves to domestic economic releases in this study, Connolly and Kohler (2004) find that foreign market movements represented in changes in United States interest rate futures prices are also important in explaining changes in interest rate expectations for Australia.

$$REconomic_t^n = \frac{\sum_{r=t}^{T_1} EIR_r^n}{\sum_{r=T_0+1}^{T_1} EIR_r^n} \tag{12}$$

where $EIR^n_r$ is the number of separate types of economic releases on day $r$ between the expiration of the previous futures contract at $T_0$ and the expiration of the current near futures contract at $T_1$.

A percentage bid-ask spread (*BAS*) is computed for every quotation as: $BAS = [(ask - bid)/(bid + ask)/2]$. We partition each day into 24 successive 15-minute intervals. Following McInish and Wood (1992), we calculate time-weighted percentage bid-ask spreads for both stocks and futures in each time interval. The time-weighting is based on the number of seconds the quotation is outstanding during the 15-minute interval as follows:

$$Spread_t = \frac{\sum_{i=1}^{n} BAS_i t_i}{\sum_{i=1}^{n} t_i} \tag{13}$$

where

$BAS_i =$ the percentage quoted bid-ask spread;
$t_i =$ the length of time that spread $i$ is outstanding; and
$n =$ the number of different bid-ask spreads that occur during interval $j$.

In the case of the constituent stocks in the index, the time-weighted percentage bid-ask spread for each stock is further weighted according to the float-adjusted weight of the stock in the index, such that the bid-ask spreads of stocks with the greatest weight in the index have the greatest weight in the overall measure of index bid-ask spread. Finally, the percentage bid-ask spreads are averaged across the portion of the trading day for which the markets in both the futures and their underlying stocks are open simultaneously (that is, from 10.00 a.m. to 4.00 p.m.) to create a time series of daily bid-ask spreads.

Price volatility is measured as:

$$Volatility_t = \sum_{j=1}^{N_t} \left[\log H_t(j) - \log L_t(j)\right] / N_t \tag{14}$$

where $H_t(j)$ is the highest price during the $j$th quarter hour interval on day $t$ and $L_t(j)$ is the lowest price.

Both volume and open interest in number of shares or contracts are multiplied by their associated prices so that the unit of analysis is the Australian dollar value of turnover ($Turnover_t$) or open interest ($OpenInterest_t$).

All variables (futures contract mispricing, dividend yield uncertainty, dividend announcements, interest rate options implied volatility, economic releases, bid-ask spreads, price volatility, turnover and open interest) are transformed into standardised deviates. The variables for the stock index or futures market are standardised by subtracting the mean for the quarterly expiry cycle and dividing by the standard deviation for the quarterly expiry cycle for the respective stock index or futures. We need to control for differences in mispricing, risk levels, transaction costs, volatility and liquidity across contracts[6].

## 3. Behaviour of the mispricing series

The mispricing levels for each contract are shown in table 1. Over the 16 contracts, the average mispricing is -0.03 percent[7]. The average mispricing is the lowest for the June 2002 contract (-0.15 percent); it is highest for the March and September 2003 contracts (0.03 percent). The average mispricing is lower than the estimate provided by MacKinlay and Ramaswamy (1988) of 0.12 percent for near S&P 500 futures contracts and higher than the estimate provided by Yadav and Pope (1994) of -0.15 percent for near FTSE 100 stock index futures. The overall standard deviation of the mispricing levels is 0.10 percent.

The average of the absolute mispricing for the near contract is 0.09 percent[8].

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[6] We performed the same analysis with the raw data and found the results to be similar to those obtained using the standardised measures.

[7] When measured as the difference between the actual and theoretical index futures contract price, the mean mispricing over the sample period is 1.3 index points where each index point is valued at AUD 25.

[8] When measured as the absolute difference between the actual and theoretical index futures contract price, the mean absolute mispricing over the sample period is 3.2 index points where each index point is valued at AUD 25.

## Table 1

**Summary statistics on the levels of mispricing in SFE SPI 200[TM] Index Futures contracts, by expiration (15-minute quote snapshot data, mispricing in percent of index value)**

| | $M^n_t$ | | | | | | $\lvert M^n_t \rvert$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Assuming cash is fully valued and franking credits are worthless ($\gamma_1$=1 and $\gamma_2$=0) | | Assuming cash and franking credits are partially valued ($\gamma_1$=0.80 and $\gamma_2$=0.57) | | Assuming cash and franking credits are fully valued ($\gamma_1$=$\gamma_2$=1) | | Assuming cash is fully valued and franking credits are worthless ($\gamma_1$=1 and $\gamma_2$=0) | | Assuming cash and franking credits are partially valued ($\gamma_1$=0.80 and $\gamma_2$=0.57) | | Assuming cash and franking credits are fully valued ($\gamma_1$=$\gamma_2$=1) | | |
| | Mean % | Std. dev. % | Mean % | Std. dev. % | Mean % | Std. dev. % | Mean % | Std. dev. % | Mean % | Std. dev. % | Mean % | Std. dev. % | *N* |
| **Contract** | | | | | | | | | | | | | |
| Mar-02 | 0.03 | 0.11 | 0.02 | 0.10 | 0.25 | 0.17 | 0.10 | 0.06 | 0.10 | 0.06 | 0.26 | 0.15 | 60 |
| Jun-02 | -0.14 | 0.10 | -0.15 | 0.10 | 0.01 | 0.12 | 0.14 | 0.09 | 0.16 | 0.09 | 0.11 | 0.06 | 63 |
| Sep-02 | -0.07 | 0.08 | -0.08 | 0.08 | 0.17 | 0.13 | 0.09 | 0.05 | 0.11 | 0.06 | 0.19 | 0.10 | 65 |
| Dec-02 | 0.00 | 0.07 | 0.00 | 0.08 | 0.15 | 0.13 | 0.07 | 0.04 | 0.07 | 0.04 | 0.17 | 0.10 | 58 |
| Mar-03 | 0.12 | 0.17 | 0.03 | 0.09 | 0.30 | 0.16 | 0.14 | 0.16 | 0.08 | 0.05 | 0.30 | 0.16 | 68 |
| Jun-03 | 0.01 | 0.13 | 0.01 | 0.13 | 0.20 | 0.14 | 0.08 | 0.10 | 0.08 | 0.10 | 0.22 | 0.11 | 52 |
| Sep-03 | 0.05 | 0.07 | 0.03 | 0.06 | 0.28 | 0.15 | 0.08 | 0.05 | 0.06 | 0.03 | 0.29 | 0.14 | 65 |
| Dec-03 | -0.08 | 0.09 | -0.07 | 0.08 | 0.10 | 0.12 | 0.10 | 0.07 | 0.09 | 0.06 | 0.13 | 0.10 | 64 |
| Mar-04 | -0.04 | 0.08 | -0.05 | 0.07 | 0.15 | 0.12 | 0.08 | 0.04 | 0.08 | 0.04 | 0.17 | 0.10 | 61 |
| Jun-04 | -0.03 | 0.11 | -0.02 | 0.10 | 0.17 | 0.10 | 0.09 | 0.06 | 0.09 | 0.06 | 0.18 | 0.10 | 61 |
| Sep-04 | 0.03 | 0.09 | 0.01 | 0.09 | 0.26 | 0.14 | 0.08 | 0.05 | 0.08 | 0.05 | 0.26 | 0.14 | 66 |
| Dec-04 | -0.02 | 0.07 | -0.01 | 0.07 | 0.14 | 0.11 | 0.07 | 0.04 | 0.07 | 0.04 | 0.15 | 0.10 | 63 |
| Mar-05 | 0.00 | 0.11 | -0.03 | 0.10 | 0.19 | 0.16 | 0.09 | 0.06 | 0.09 | 0.05 | 0.20 | 0.15 | 61 |
| Jun-05 | -0.04 | 0.07 | -0.04 | 0.07 | 0.13 | 0.07 | 0.07 | 0.04 | 0.07 | 0.05 | 0.13 | 0.07 | 61 |
| Sep-05 | -0.03 | 0.09 | -0.09 | 0.08 | 0.16 | 0.13 | 0.08 | 0.05 | 0.10 | 0.07 | 0.17 | 0.11 | 65 |
| Dec-05 | -0.08 | 0.10 | -0.10 | 0.11 | 0.05 | 0.10 | 0.11 | 0.07 | 0.12 | 0.08 | 0.09 | 0.07 | 65 |
| **Overall** | -0.02 | 0.11 | -0.03 | 0.10 | 0.17 | 0.15 | 0.09 | 0.07 | 0.09 | 0.06 | 0.19 | 0.13 | 998 |

Mispricing = futures bid-ask midpoint price - theoretical price

The first order autocorrelations in the mispricing series are negative for 14 out 16 contracts as shown in table 2, suggesting that the day-to-day levels of mispricing are mean reverting. Any substantial deviations are inclined to be arbitraged away from one day to the next.

## Table 2

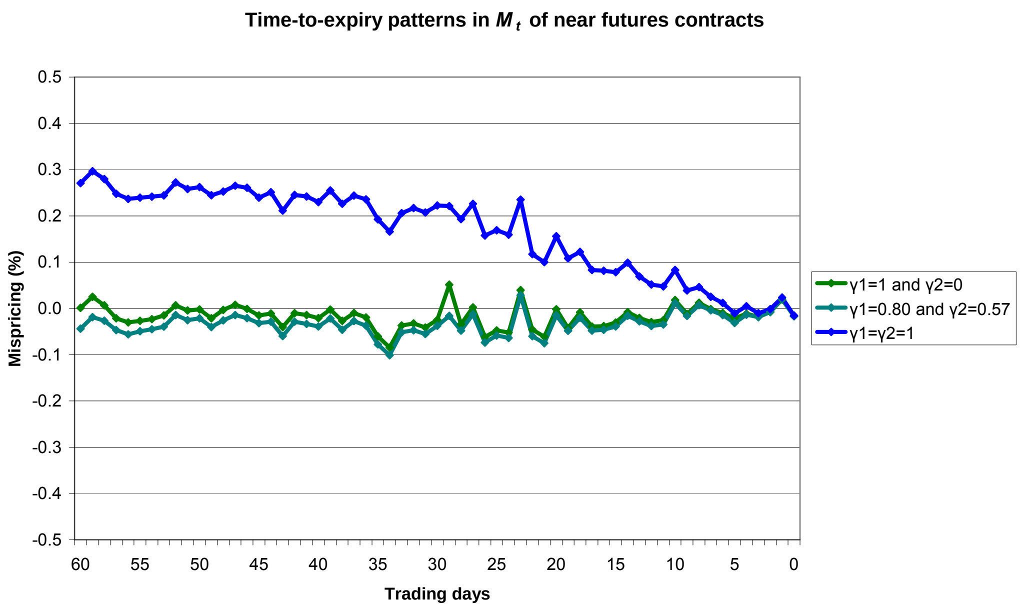
**First order autocorrelations for near futures contract standardised mispricing by contract**

| | $M^n_t$ | | | | | | $\lvert M^n_t \rvert$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Assuming cash is fully valued and franking credits are worthless ($\gamma_1$=1 and $\gamma_2$=0) | | Assuming cash and franking credits are partially valued ($\gamma_1$=0.80 and $\gamma_2$=0.57) | | Assuming cash and franking credits are fully valued ($\gamma_1$=$\gamma_2$=1) | | Assuming cash is fully valued and franking credits are worthless ($\gamma_1$=1 and $\gamma_2$=0) | | Assuming cash and franking credits are partially valued ($\gamma_1$=0.80 and $\gamma_2$=0.57) | | Assuming cash and franking credits are fully valued ($\gamma_1$=$\gamma_2$=1) | |
| | | $\lvert t \rvert$ | | $\lvert t \rvert$ | | $\lvert t \rvert$ | | $\lvert t \rvert$ | | $\lvert t \rvert$ | | $\lvert t \rvert$ |
| **Contract** | | | | | | | | | | | | |
| Mar-02 | 0.03 | 0.22 | 0.00 | 0.03 | 0.03 | 0.22 | 0.14 | 1.01 | 0.12 | 0.83 | -0.03 | 0.24 |
| Jun-02 | -0.07 | 0.54 | -0.27 | 2.03 | -0.20 | 1.44 | -0.08 | 0.57 | -0.26 | 1.91 | -0.23 | 1.68 |
| Sep-02 | -0.24 | 1.79 | -0.24 | 1.79 | -0.06 | 0.47 | -0.20 | 1.50 | -0.30 | 2.26 | -0.04 | 0.27 |
| Dec-02 | -0.20 | 1.45 | -0.19 | 1.38 | -0.10 | 0.72 | -0.08 | 0.59 | -0.05 | 0.35 | -0.11 | 0.80 |
| Mar-03 | -0.38 | 2.90 | -0.34 | 2.61 | -0.25 | 1.86 | -0.38 | 2.95 | -0.34 | 2.61 | -0.01 | 0.06 |
| Jun-03 | -0.17 | 1.12 | -0.18 | 1.17 | -0.25 | 1.67 | -0.10 | 0.68 | -0.12 | 0.76 | -0.22 | 1.46 |
| Sep-03 | -0.37 | 2.87 | -0.37 | 2.91 | -0.39 | 3.09 | -0.43 | 3.48 | -0.39 | 3.02 | -0.40 | 3.13 |
| Dec-03 | 0.00 | 0.03 | -0.02 | 0.15 | -0.04 | 0.31 | 0.10 | 0.69 | 0.10 | 0.68 | -0.04 | 0.25 |
| Mar-04 | -0.13 | 0.93 | -0.15 | 1.08 | -0.21 | 1.54 | 0.04 | 0.28 | -0.09 | 0.62 | -0.24 | 1.75 |
| Jun-04 | -0.14 | 1.02 | -0.13 | 0.95 | -0.28 | 2.16 | -0.15 | 1.07 | -0.14 | 1.04 | -0.26 | 1.95 |
| Sep-04 | -0.25 | 1.88 | -0.28 | 2.11 | -0.17 | 1.22 | -0.06 | 0.43 | 0.07 | 0.52 | -0.16 | 1.20 |
| Dec-04 | -0.26 | 1.96 | -0.25 | 1.91 | -0.23 | 1.72 | -0.09 | 0.68 | -0.25 | 1.91 | -0.15 | 1.11 |
| Mar-05 | 0.13 | 0.95 | 0.14 | 0.97 | 0.19 | 1.39 | 0.25 | 1.81 | 0.23 | 1.69 | 0.23 | 1.68 |
| Jun-05 | -0.02 | 0.15 | -0.01 | 0.04 | 0.04 | 0.26 | 0.03 | 0.18 | 0.03 | 0.25 | 0.08 | 0.59 |
| Sep-05 | -0.19 | 1.37 | -0.17 | 1.27 | -0.20 | 1.46 | -0.26 | 1.89 | -0.16 | 1.17 | -0.26 | 1.95 |
| Dec-05 | -0.17 | 1.24 | -0.14 | 0.99 | -0.20 | 1.47 | -0.20 | 1.48 | -0.27 | 2.05 | -0.06 | 0.44 |

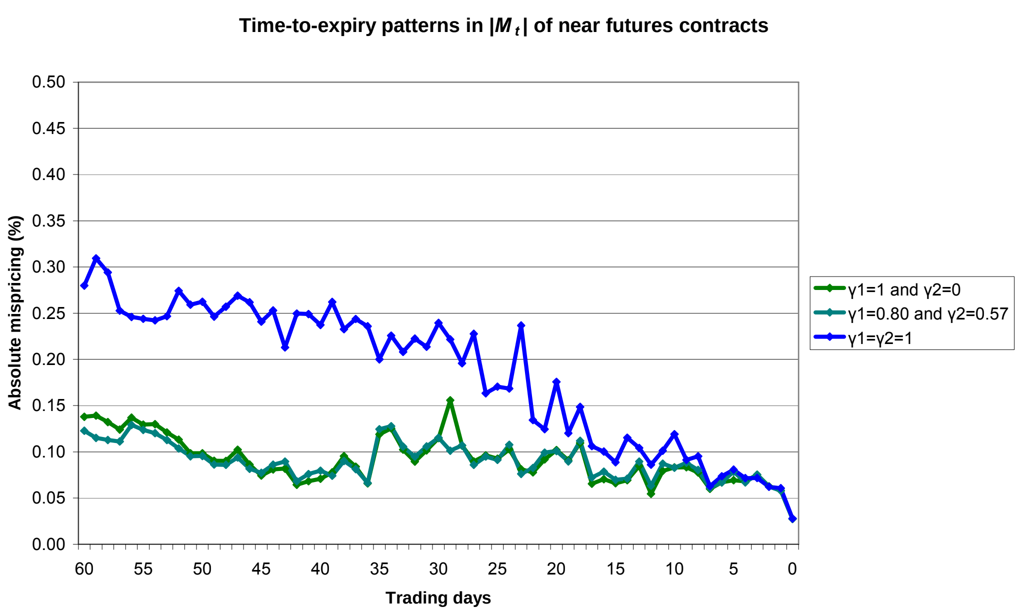
Autocorrelations are based on daily average observation intervals.

The relationship between the level of the mispricing and the time to maturity of the contract is shown in figure 1 panel A.

**Figure 1**
**Panel A**




**Panel B**




We test for the strength of the time to maturity relationship with the model

$$STM_t^n = \alpha_0 + \sum_{i=1}^{n} \alpha_i D_i + \varepsilon_t \text{, } i = 1, \ldots, 6 \tag{15}$$

where

$STM_t^n \equiv$ standardised mispricing (the average intraday percentage mispricing, scaled by subtracting the mean for the contract and dividing by the standard deviation for the contract on which mispricing $t$ is observed) of the near futures index contract on day $t$;

$D_i \equiv$ fortnightly dummy variables designed to capture the pattern in mispricing related to the time to expiration of the near contract (e.g., $D_1 = 1$ where $i = 1$ to 10 trading days prior to expiration of the near contract, else 0; $D_2 = 1$ where $i = 11$ to 20 else 0; etc.; and $D_6 = 1$ where $i = 51$ to 60 else 0).

The generalised method of moments is used to improve the efficiency of the parameter estimation in the presence of heteroskedasticity and autocorrelation of unknown forms. The Bartlett kernel with bandwidth parameter $l(n) = 7$ is used to estimate consistent covariance matrices of the parameter estimates as outlined by Newey and West (1987). This bandwidth value corresponds to the smallest lag selection parameter $n = [4(T/100)^{2/9}]$ proposed by Newey and West (1994), taking into account the degree of first order autocorrelation in the residuals and the size of our sample $T$ (Andews, 1991). All $t$-statistics are adjusted accordingly.

Table 3 displays the results. Average mispricing is significantly higher in the last fortnight before expiry than in the third last fortnight (represented in the constant term $\alpha_0$). However, there does not appear to be any other systematic trend in the mispricing for longer times to maturity.

**Table 3**

**Time-to-expiry patterns in near futures contract standardised mispricing**

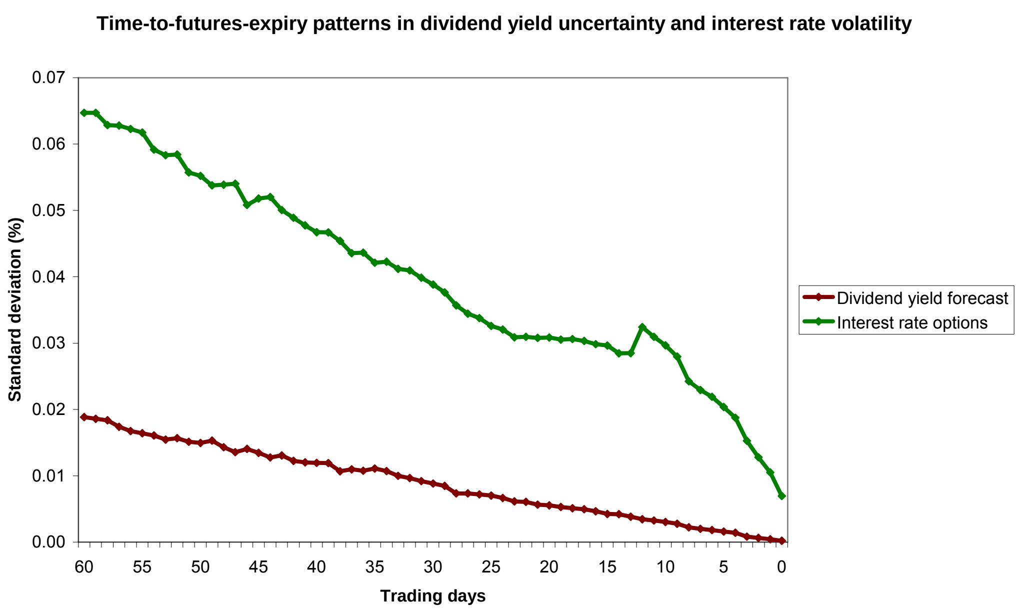
| | Assuming cash is fully valued and franking credits are worthless ($\gamma_1$=1 and $\gamma_2$=0) | | | | Assuming cash and franking credits are partially valued ($\gamma_1$=0.80 and $\gamma_2$=0.57) | | | | Assuming cash and franking credits are fully valued ($\gamma_1$=$\gamma_2$=1) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $STM^n_t$ | \|$t$\| | $ST\|M^n_t\|$ | \|$t$\| | $STM^n_t$ | \|$t$\| | $ST\|M^n_t\|$ | \|$t$\| | $STM^n_t$ | \|$t$\| | $ST\|M^n_t\|$ | \|$t$\| |
| **Time-to-expiry patterns** | | | | | | | | | | | | |
| $\alpha_0$ | -0.09 | 0.66 | 0.12 | 1.19 | -0.08 | 0.60 | 0.14 | 1.26 | 0.08 | 0.90 | 0.03 | 0.28 |
| $D_1$ | 0.27 | 1.58 | -0.48 | 3.95* | 0.38 | 2.26* | -0.49 | 3.93* | -1.20 | 9.93* | -0.92 | 6.64* |
| $D_2$ | 0.04 | 0.22 | -0.30 | 2.34* | 0.06 | 0.36 | -0.25 | 1.86 | -0.67 | 5.13* | -0.63 | 4.89* |
| $D_4$ | -0.05 | 0.32 | -0.13 | 0.94 | -0.07 | 0.48 | -0.11 | 0.75 | 0.28 | 2.43* | 0.32 | 2.74* |
| $D_5$ | 0.17 | 1.01 | -0.26 | 1.75 | 0.15 | 0.88 | -0.27 | 1.74 | 0.49 | 3.83* | 0.51 | 3.75* |
| $D_6$ | 0.10 | 0.42 | 0.39 | 2.28* | 0.03 | 0.12 | 0.25 | 1.45 | 0.56 | 3.97* | 0.55 | 3.52* |
| adj $R^2$ | 0.01 | | 0.07 | | 0.02 | | 0.05 | | 0.43 | | 0.34 | |
| $F$ | 2.25* | | 15.64* | | 4.12* | | 11.33* | | 143.26* | | 94.80* | |
| $N$ | 930 | | 930 | | 930 | | 930 | | 930 | | 930 | |

The table presents the results of the regression equation $STM^n_t = \alpha_0 + \sum\alpha_i D_i + \varepsilon_t$ where $D_i$ is a time-to-expiry dummy variable ($ST|M^n_t|$ is the dependent variable in the second, fourth and sixth columns). The standardised variable is defined as $(X_{it} - \mu_i)/S_i$ where $X_{it}$ is the raw variable, $\mu_i$ is the mean for the quarterly expiry cycle and $S_i$ is the standard deviation for the quarterly expiry cycle. *Denotes significance at the 5% level.

The relationship between time-to-expiry and the average absolute mispricing is depicted in figure 1 panel B. Given the estimate of the value placed on the cash dividends ($\gamma_1 = 0.800$) and the imputation tax credits ($\gamma_2 = 0.572$), the mispricing of the near contract decreases over the first month of the expiry cycle (when there is longer than 40 trading days to expiry), is somewhat constant across the second month (between 40 and 20 trading days prior to expiry) and decreases again over the last month (less than 20 trading days remaining)[9]. Average absolute mispricing is significantly higher in the last fortnight before expiry than in the third last fortnight, as shown in table 3. While the average absolute mispricing is higher at the beginning of the expiry cycle (six fortnights from expiry), the coefficient on $D_6$ is insignificant at the 5 percent level. The $R^2$ statistic of 0.05 suggests either that only a small portion of the absolute mispricing is explained by time to expiration effects or that countervailing forces act to nullify these effects. The relationship between the absolute value of mispricing and time to expiration is considerably weaker than found in previous research that shows a strong and statistically significant time to expiration pattern (MacKinlay and Ramaswamy, 1988; Yadav and Pope, 1994).

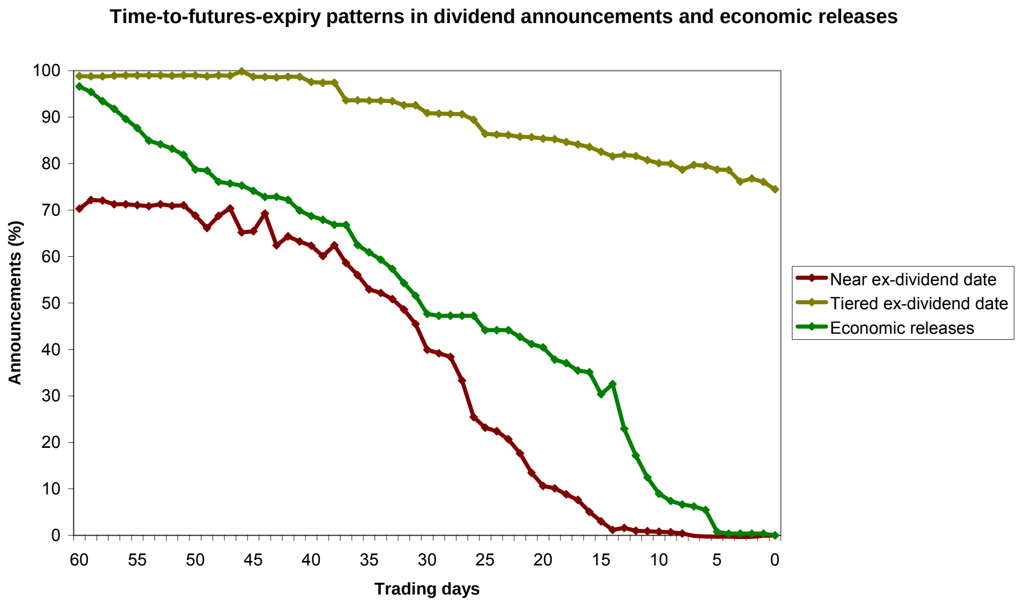
The varying speeds at which the dividend yield and interest rate uncertainties inherent in index arbitrage activity are resolved across the expiry cycle are shown in figures 2 and 3. Figure 2 illustrates the risks surrounding the magnitude of the valuation parameters in the cost of carry model: the weighted average standard deviation of analysts' dividend forecasts for the stocks underlying the index and the volatility implied by interest rate option prices. Figure 3 shows the risks associated with the timing of announcements that resolve the uncertainty over the valuation parameters: the amount of gross dividends announced over the remaining life of the futures contract with ex-dividend dates before and after contract expiry and the number of economic releases likely to influence interest rate expectations over the remaining life of the contract.

**Figure 2**




[9] This result is sensitive to the assumption about the value placed on the tax credits. Under the most liberal assumption that the tax credits are fully valued ($\gamma_1 = \gamma_2 = 1$), the decrease in mispricing accelerates considerably between 40 and 20 trading days prior to expiry.

**Figure 3**




In contrast to the rapidly declining risks for index arbitrage, the price volatility and bid-ask spreads are largely constant across the expiry cycle, both for the index futures and their underlying stocks as shown in figure 4. Futures prices are slightly more volatile than for the index itself in terms of the average 15-minute price range, which is consistent with prior research (MacKinlay and Ramaswamy, 1988). There does not appear to be any drift in the width of the arbitrage window due to systematic changes in transaction costs for the near index futures and their underlying stocks across the expiry cycle.

**Figure 4**

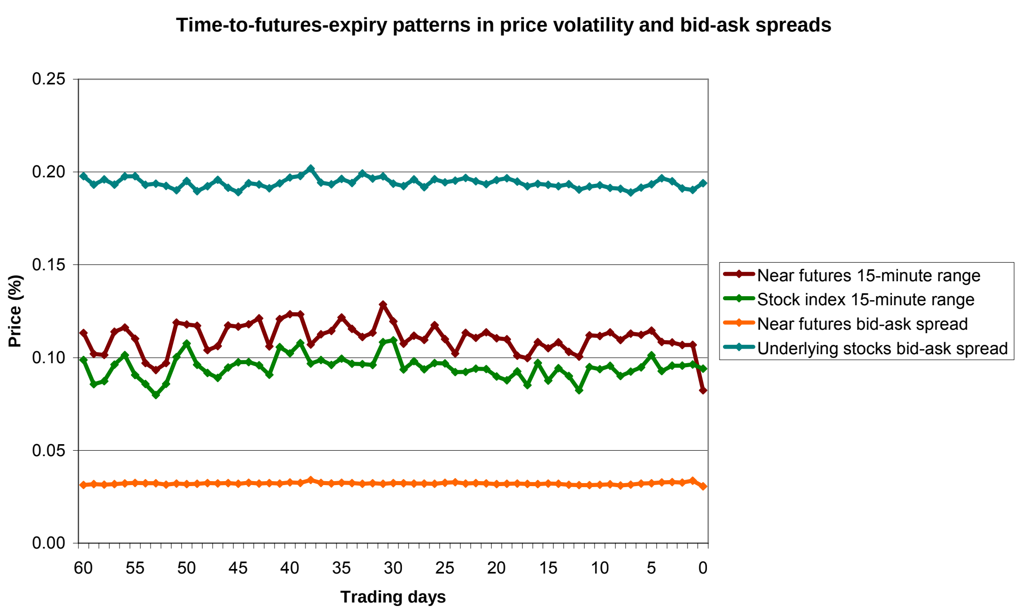


Figure 5 illustrates the daily changes in volume and open interest in near SFE SPI 200™ futures and the trading activity in the underlying stocks leading up to the expiration of the futures contract. The turnover of the near futures contract increases rapidly from 8 trading days before the expiry date and peaks on the third last trading day (Tuesday). Open interest for the near futures contract builds up steadily across the expiry cycle and then dissipates rapidly from the last Tuesday onwards. Trading activity in the underlying stocks also responds to the futures expiration cycle. The turnover of constituent stocks in the S&P/ASX 200 index peaks on the last trading day in the futures expiration cycle. The implementation of index arbitrage strategies appears to drive large increases in trading activity for the underlying stocks on the day that the futures expire.

**Figure 5**

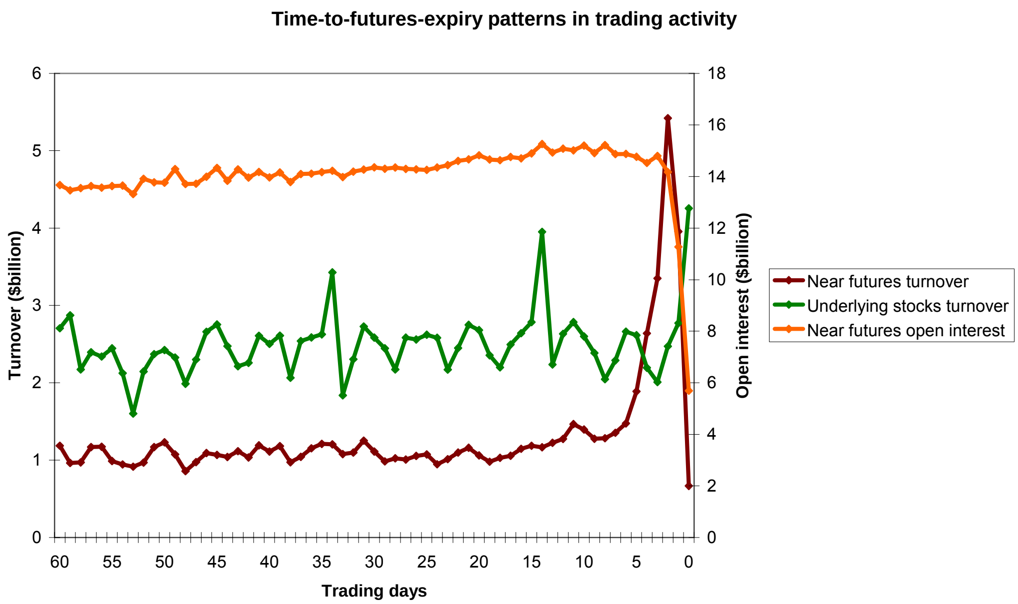



Table 4 confirms that strong time to expiration patterns are present in the risk measurements: the dispersion of analysts' dividend yield forecasts, the gross dividend amounts announced over the remaining life of the futures contract, the implied volatility of interest rate options and the number of economic releases due before expiration. The coefficients for all of the time-to-expiry dummy variables when modelling these risks are highly statistically significant and consistent with the presence of greater arbitrage risks for longer times to maturity. The table confirms that the price volatilities and bid-ask spreads for the futures and their underlying stocks are unrelated to the time to expiration of the futures, except that the bid-ask spreads in the futures market are significantly lower in the second last fortnight prior to maturity. The table also confirms the time to expiration pattern in turnover for the near futures contract ($D_1$, $D_2$ and $D_4$ are positive and significant and the constant term is negative and significant). The coefficient on $D_2$ and the constant term are positive and significant and the coefficients on $D_5$ and $D_6$ are negative and significant for the standardised open interest variable for the near futures contract, confirming that open interest in the near contract increases steadily across the expiry cycle.

# Table 4

**Time-to-futures-expiry patterns in standardised dividend yield uncertainty, interest rate volatility, bid-ask spreads, price volatility and trading activity**

| | Dividend yield forecast standard deviation | | Gross dividend amount | | | | Interest rate options implied volatility | | Economic releases | | Bid-ask spread | | | | 15-minute price range | | | | Turnover | | | | Near futures open interest | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Near ex-date | | Tiered ex-date | | | | | | Index constituent stocks | | Near futures | | Index constituent stocks | | Near futures | | Index constituent stocks | | Near futures | | | |
| | | $\|t\|$ | | $\|t\|$ | | $\|t\|$ | | $\|t\|$ | | $\|t\|$ | | $\|t\|$ | | $\|t\|$ | | $\|t\|$ | | $\|t\|$ | | $\|t\|$ | | $\|t\|$ | | $\|t\|$ |
| **Time-to-expiry patterns** | | | | | | | | | | | | | | | | | | | | | | | | |
| $\alpha_0$ | -0.37 | 9.22* | -0.37 | 5.17* | -0.04 | 0.46 | -0.42 | 7.81* | -0.20 | 8.45* | -0.02 | 0.16 | 0.11 | 1.09 | 0.09 | 0.87 | 0.10 | 1.08 | -0.00 | 0.06 | -0.31 | 10.21* | 0.17 | 2.16* |
| $D_1$ | -0.92 | 15.73* | -0.86 | 10.64* | -1.26 | 9.34* | -0.75 | 6.58* | -1.35 | 37.36* | -0.25 | 1.60 | -0.16 | 0.96 | -0.08 | 0.58 | -0.09 | 0.61 | -0.10 | 1.00 | 1.69 | 8.54* | 0.11 | 0.61 |
| $D_2$ | -0.45 | 8.16* | -0.71 | 8.98* | -0.49 | 3.98* | -0.22 | 2.81* | -0.49 | 7.66* | -0.04 | 0.36 | -0.38 | 2.94* | -0.26 | 1.95 | -0.22 | 1.76 | 0.23 | 1.96 | 0.16 | 3.34* | 0.32 | 3.23* |
| $D_4$ | 0.63 | 9.34* | 0.93 | 9.95* | 0.46 | 4.27* | 0.56 | 8.15* | 0.53 | 11.16* | 0.18 | 1.16 | 0.09 | 0.64 | 0.14 | 0.99 | 0.15 | 1.14 | 0.04 | 0.38 | 0.11 | 2.38* | -0.15 | 1.54 |
| $D_5$ | 1.09 | 16.33* | 1.30 | 16.41* | 0.80 | 7.83* | 1.09 | 14.00* | 0.94 | 24.25* | -0.13 | 0.78 | -0.04 | 0.26 | -0.09 | 0.47 | -0.04 | 0.20 | -0.13 | 1.10 | 0.06 | 1.01 | -0.24 | 2.30* |
| $D_6$ | 1.64 | 28.14* | 1.46 | 17.06* | 0.83 | 8.35* | 1.64 | 22.48* | 1.39 | 35.13* | 0.23 | 1.17 | -0.01 | 0.06 | -0.14 | 0.86 | -0.20 | 1.32 | -0.24 | 1.78 | -0.00 | 0.02 | -0.45 | 4.40* |
| adj $R^2$ | 0.89 | | 0.90 | | 0.66 | | 0.78 | | 0.93 | | 0.02 | | 0.02 | | 0.01 | | 0.01 | | 0.02 | | 0.36 | | 0.11 | |
| $F$ | 1,466.63* | | 1,703.54* | | 365.30* | | 657.16* | | 2,398.62* | | 5.42* | | 4.77* | | 2.96* | | 3.03* | | 4.78* | | 105.73* | | 23.73* | |
| $N$ | 933 | | 933 | | 933 | | 929 | | 933 | | 933 | | 933 | | 933 | | 933 | | 933 | | 934 | | 925 | |

The table presents the results of the regression equation $STV_t = \alpha_0 + \sum\alpha_i D_i + \varepsilon_t$ where $V_t$ is dividend yield forecast standard deviation, gross dividend amount, interest rate options implied volatility, economic releases, bid-ask spread, 15-minute price range, turnover or open interest and $D_i$ is a time-to-expiry dummy variable. The standardised variable is defined as $(X_{it} - \mu_i)/S_i$ where $X_{it}$ is the raw variable, $\mu_i$ is the mean for the quarterly expiry cycle and $S_i$ is the standard deviation for the quarterly expiry cycle.
*Denotes significance at the 5% level.

We proceed to supplement our model for the absolute mispricing of the index futures as a function involving the various forms of uncertainty surrounding the valuation parameters, as well as transaction costs, volatility and trading activity. The full model taking each of these additional factors into account is:

$$\begin{aligned} ST\left|M_t^n\right| = \quad & \alpha_0' + \beta_1 STUDividend_t^n + \beta_2 STADividend_t^n + \beta_3 STADividend_t^s \\ & + \beta_4 STIVInterest_t^n + \beta_5 STREconomic_t^n \\ & + \beta_6 STSpread_t^c + \beta_7 STSpread_t^n + \beta_8 STVolatility_t^c \\ & + \beta_9 STTurnover_t^n + \beta_{10} STOpenInterest_t^n + \sum_{i=1}^{n} \alpha_i' D_i + \varepsilon_t \end{aligned} \tag{16}$$

where all the variables remain as defined above.

Table 5 shows the estimated results from a regression of the standardised near futures contract absolute mispricing on the time to expiration dummy variables after controlling for the timing of dividend announcements with ex-dividend dates before contract expiry and the price volatility of the index (model 1), as well as the dispersion of analysts' dividend yield forecasts, the timing of dividend announcements with ex-dividend dates after contract expiry, the implied volatility of interest rate options, bid-ask spreads and trading activity (model 2). Three results emerge from the table. First, the coefficients on the standardised proportion of dividend announcements over the remaining life of the contract with ex-dividend dates before contract expiry ($STADividend^n{}_t$) and the standardised volatility of the index ($STVolatility^c{}_t$) are positive and significant and the $F$-statistics for both models are significant at the 5 percent level. Second, the results for model 2 imply that after controlling for the dispersion of analysts' dividend yield forecasts, the timing of dividend announcments with ex-dividend dates after contract expiration, the interest rate options implied volatility, bid-ask spreads and trading activity, the time-to-expiry dummy variable $D_1$ is no longer significant whereas $D_4$ and $D_5$ become significant. This indicates that the time to expiration pattern in absolute mispricing unexplained by our extended model incorporating risky arbitrage is confined to a reduction in absolute mispricing levels for longer times to maturity (particularly during the second and third fortnights at the beginning of the expiry cycle). Third, the results for model 2 indicate that the dispersion of analysts' dividend yield forecasts, the timing of dividend announcements with ex-dividend dates after contract expiration, the implied volatility of interest rate options, bid-ask spreads and trading activity provide little incremental explanatory power and are not significantly related to near futures contract mispricing. These results suggest that a decrease in the magnitude of near futures contract mispricing across the expiry cycle is related to the timing of dividend announcements and the price volatility of the index. In addition, there appear to be countervailing forces possibly including arbitrageurs' early unwinding option and rollover option that concurrently serve to reduce the width of the arbitrage window for longer times to maturity.

## Table 5

**Relationship between mispricing of near contracts and valuation parameter uncertainty**

| | Assuming cash is fully valued and franking credits are worthless ($\gamma_1$=1 and $\gamma_2$=0) | | | | Assuming cash and franking credits are partially valued ($\gamma_1$=0.80 and $\gamma_2$=0.57) | | | | Assuming cash and franking credits are fully valued ($\gamma_1$=$\gamma_2$=1) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Model 1 | | Model 2 | | Model 1 | | Model 2 | | Model 1 | | Model 2 | |
| | $ST\|M^n_t\|$ | $\|t\|$ | $ST\|M^n_t\|$ | $\|t\|$ | $ST\|M^n_t\|$ | $\|t\|$ | $ST\|M^n_t\|$ | $\|t\|$ | $ST\|M^n_t\|$ | $\|t\|$ | $ST\|M^n_t\|$ | $\|t\|$ |
| **Variable** | | | | | | | | | | | | |
| $\alpha_0$ | 0.21 | 1.84 | 0.17 | 1.33 | 0.24 | 2.16* | 0.21 | 1.68 | 0.16 | 1.68 | 0.12 | 1.15 |
| $STUDividend^n_t$ | - | - | 0.00 | 0.02 | - | - | -0.07 | 0.55 | - | - | 0.03 | 0.27 |
| $STADividend^n_t$ | 0.26 | 1.97* | 0.24 | 1.81 | 0.33 | 2.49* | 0.31 | 2.36* | 0.18 | 1.46 | 0.20 | 1.66 |
| $STADividend^s_t$ | - | - | -0.08 | 1.07 | - | - | -0.05 | 0.76 | 0.24 | 2.76* | 0.23 | 2.68* |
| $STIVInterest^n_t$ | - | - | 0.00 | 0.03 | - | - | 0.05 | 0.63 | - | - | -0.11 | 1.17 |
| $STREconomic^n_t$ | - | - | - | - | - | - | - | - | 0.38 | 3.08* | 0.35 | 2.61* |
| $STSpread^c_t$ | - | - | -0.05 | 1.31 | - | - | -0.06 | 1.51 | - | - | 0.03 | 0.81 |
| $STSpread^n_t$ | - | - | 0.02 | 0.77 | - | - | -0.00 | 0.09 | - | - | -0.01 | 0.30 |
| $STVolatility^c_t$ | 0.17 | 4.38* | 0.20 | 4.81* | 0.18 | 5.08* | 0.22 | 5.61* | - | - | -0.04 | 1.25 |
| $STTurnover^n_t$ | - | - | -0.06 | 1.72 | - | - | -0.05 | 1.32 | - | - | -0.04 | 1.27 |
| $STOpenInterest^n_t$ | - | - | 0.03 | 0.68 | - | - | 0.02 | 0.38 | 0.09 | 2.51* | 0.10 | 2.64* |
| $D_1$ | -0.24 | 1.53 | -0.28 | 1.40 | -0.20 | 1.24 | -0.25 | 1.26 | 0.04 | 0.18 | 0.05 | 0.23 |
| $D_2$ | -0.07 | 0.48 | -0.10 | 0.63 | 0.03 | 0.18 | -0.02 | 0.12 | -0.23 | 1.58 | -0.24 | 1.64 |
| $D_4$ | -0.39 | 2.15* | -0.33 | 1.72 | -0.43 | 2.42* | -0.37 | 1.98* | -0.13 | 0.80 | -0.09 | 0.54 |
| $D_5$ | -0.58 | 2.59* | -0.50 | 1.89 | -0.68 | 2.98* | -0.59 | 2.26* | -0.25 | 1.19 | -0.17 | 0.74 |
| $D_6$ | 0.04 | 0.14 | 0.11 | 0.33 | -0.20 | 0.76 | -0.12 | 0.37 | -0.36 | 1.50 | -0.24 | 0.90 |
| adj $R^2$ | 0.11 | | 0.10 | | 0.10 | | 0.10 | | 0.38 | | 0.38 | |
| $F$ | 16.82* | | 8.49* | | 15.31* | | 7.91* | | 64.31* | | 38.46* | |
| $N$ | 930 | | 918 | | 930 | | 918 | | 923 | | 918 | |

The table presents the results of the regression equation:

$$\begin{aligned} ST\left|M^n_t\right| = \; & \alpha'_0 + \beta_1 STUDividend^n_t + \beta_2 STADividend^n_t + \beta_3 STADividend^s_t \\ & + \beta_4 STIVInterest^n_t + \beta_5 STREconomic^n_t + \beta_6 STSpread^c_t + \beta_7 STSpread^n_t \\ & + \beta_8 STVolatility^c_t + \beta_9 STTurnover^n_t + \beta_{10} STOpenInterest^n_t + \sum_{i=1}^{n} \alpha'_i D_i + \varepsilon_t \end{aligned}$$

where $M^n_t$ is the mispricing of the near futures contract, $UDividend^n_t$ is the uncertainty about dividends paid by underlying stocks over the remaining life of the contract, $ADividend^n_t$ is the magnitude of gross dividends announced over the remaining life of the contract with ex-dividend dates before the contract expires, $ADividend^s_t$ is the magnitude of gross dividends announced over the remaining life of the contract with ex-dividend dates after the contract expires, $IVInterest^n_t$ is the implied volatility of interest rate options, $REconomic^n_t$ is number of economic releases over the remaining life of the contract, $Spread^c_t$ is the time-weighted average bid-ask spread for index constituent stocks, $Spread^n_t$ is the time-weighted average bid-ask spread for the near futures contract, $Volatility^c_t$ is stock index volatility, $Turnover^n_t$ is notional turnover in the near futures contract, $OpenInterest^n_t$ is open interest in the near futures contract and $D_i$ is a time-to-expiry dummy variable.

The standardised variable is defined as $(X_{it} - \mu_i)/S_i$ where $X_{it}$ is the raw variable, $\mu_i$ is the mean for the quarterly expiry cycle and $S_i$ is the standard deviation for the quarterly expiry cycle. *Denotes significance at the 5% level.

## 4. Conclusion

We measure the time-dependent risks associated with implementing arbitrage strategies involving stock index futures, focussing on the uncertainty surrounding the valuation parameters in the cost of carry model. Our risk measurements reflect uncertainty relating to the timing of dividend announcements and economic releases likely to influence interest rate expectations as well as uncertainty about the level of dividend yields and interest rates up to the expiry of a futures contract. We also measure the mispricing and absolute mispricing of stock index futures taking account of the value that investors place on the cash dividends and franking credits paid by the stocks underlying the index and demonstrate (in the appendix) that this is a more reasonable basis upon which to proceed in the Australian dividend imputation context than simply ignoring the franking credits and pricing cash dividends on par with capital gains. We apply our measurements using a four year sample for SFE SPI™ Index Futures traded on the Sydney Futures Exchange.

Our main findings are as follows: (i) the timing of dividend announcements up to the expiry of an index futures contract widens the arbitrage window for longer times to maturity; (ii) the arbitrage window is also wider on days when the price volatility of the index is high relative to levels experienced at other stages of the expiry cycle; (iii) in addition, there appear to be countervailing forces possibly including arbitrageurs' early unwinding option and rollover option that concurrently reduce the width of the arbitrage window for longer times to maturity (particularly for the second and third fortnights at the beginning of the expiry cycle). Future research can identify the precise nature of these countervailing forces, when controlling for the risks that we have quantified in our study.

## Appendix

## Cash dividends and franking credits

We estimate the model

$$M_t^n = \delta_0 + \beta_1 GrossDiv_t^n + \sum_{i=1}^{n} \delta_i D_i + \varepsilon_t \text{ , } i = 1, \ldots, 6 \tag{A1}$$

where

$M_t^n \equiv$ average intraday mispricing in index points of the near futures index contract on day $t$;

$D_i \equiv$ fortnightly dummy variables designed to capture the pattern in mispricing related to the time to expiration of the near contract (e.g., $D_1 = 1$ where $i = 1$ to 10 trading days prior to expiration of the near contract, else 0; $D_2 = 1$ where $i = 11$ to 20 else 0; etc.; and $D_6 = 1$ where $i = 51$ to 60 else 0).

Assume that the gross dividend is worthless. In this case, the results in table A1 imply that for every \$1 of gross dividend paid by the index constituent stocks, the futures contract is 77.0 cents under-priced. Alternatively, assume that the gross dividend is fully valued. In this case, the results in table A1 imply that for every \$1 of gross dividend paid by the underlying stocks, the futures contract is 23.0 cents overpriced. In both cases, the implication is that investors value 77.0 percent of the gross dividend on average when pricing the futures contract over the period 2002 to 2005. This is slightly higher than the gross dividend drop-off ratio of 72.4 percent estimated by Beggs and Skeels (2006) for the Australian share market from 2001 to 2004.

## Table A1

**Time-to-expiry and gross dividend drop-off patterns in near futures contract mispricing**

| | Assuming gross dividend is worthless ($\beta_1$=0) | | Assuming gross dividend is partially valued ($\beta_1$=0.77) | | Assuming gross dividend is fully valued ($\beta_1$=1) | |
|---|---|---|---|---|---|---|
| | $M^n_t$ | $\|t\|$ | $M^n_t$ | $\|t\|$ | $M^n_t$ | $\|t\|$ |
| **Panel A: Descriptive statistics** | | | | | | |
| Mean | -22.0 | | -0.7 | | 5.8 | |
| Median | -24.3 | | -0.4 | | 5.7 | |
| st. dev. | 12.7 | | 3.6 | | 5.1 | |
| $N$ | 930 | | 930 | | 930 | |
| **Panel B: Time-to-expiry and gross drop-off patterns** | | | | | | |
| $\alpha_0$ | -0.92 | 1.01 | -0.92 | 1.01 | -0.92 | 1.01 |
| $GrossDiv^n_t$ | -0.77 | 25.33* | 0.00 | 0.00 | 0.23 | 7.59* |
| $D_1$ | 0.75 | 0.82 | 0.75 | 0.82 | 0.75 | 0.82 |
| $D_2$ | 0.05 | 0.07 | 0.05 | 0.07 | 0.05 | 0.07 |
| $D_4$ | -0.11 | 0.16 | -0.11 | 0.16 | -0.11 | 0.16 |
| $D_5$ | 0.52 | 0.65 | 0.52 | 0.65 | 0.52 | 0.65 |
| $D_6$ | 0.46 | 0.47 | 0.46 | 0.47 | 0.46 | 0.47 |
| adj $R^2$ | 0.92 | | 0.00 | | 0.49 | |
| $F$ | 1,763.94* | | 1.18 | | 149.21* | |

Panel A presents some summary statistics for the average intraday mispricing of the near futures contract ($M^n_t$). Panel B presents the results of the regression equation $M^n_t = \delta_0 + \beta_1 GrossDiv^n_t + \sum \delta_i D_i + \varepsilon_t$ where $GrossDiv^n_t$ is the theoretical value of gross dividends paid out by the stocks over the remaining life of the contract defined as the difference between the theoretical price of the futures contract estimated from zero dividends and the theoretical price of the futures contract estimated using gross dividends and $D_i$ is a time-to-expiry dummy variable. *Denotes significance at the 5% level.

However, there are reasons to suspect that the market may not value equally a dollar of cash dividend and a dollar of franking credit (Beggs and Skeels, 2006). For example, foreign investors have limited ability to access Australian franking credits and these investors are likely to value franking credits less than cash dividends. Therefore, we estimate the model

$$M_t^n = \delta_0 + \gamma_1 Cash_t^n + \gamma_2 Franking_t^n + \sum_{i=1}^{n} \delta_i D_i + \varepsilon_t \text{ , } i = 1, \ldots, 6 \qquad \text{(A2)}$$

where

$M_t^n \equiv$ average intraday mispricing in index points of the near futures index contract on day $t$;

$D_i \equiv$ fortnightly dummy variables designed to capture the pattern in mispricing related to the time to expiration of the near contract (e.g., $D_1 = 1$ where $i = 1$ to 10 trading days prior to expiration of the near contract, else 0; $D_2 = 1$ where $i = 11$ to 20 else 0; etc.; and $D_6 = 1$ where $i = 51$ to 60 else 0).

Assume that both the cash and the franking credits are worthless. In this case, the results in table A2 imply that for every $1 of cash dividends paid by the underlying stocks the futures contract is 77.9 cents under-priced and for every $1 of franking credits distributed by the underlying stocks the futures contract is 71.9 cents under-priced. Alternatively, assume that both the cash and the franking credits are fully valued. In this case, the result in table A2 imply that for every $1 of cash dividends paid by the underlying stocks the futures contract is 22.1 cents overpriced and for every $1 of franking credits distributed by the underlying stocks the futures contract is 28.1 cents overpriced. In both cases, the implication is that investors value 77.9 percent of the cash dividends and 71.9 percent of the franking credits on average when pricing the futures contract over the period 2002 to 2005. This value of cash dividends is slightly lower than the cash dividend drop-off ratio of 80.0 percent and this value of franking credits is slightly higher than the franking credit drop-off ratio of 57.2 percent estimated by Beggs and Skeels (2006) for the Australian share market from 2001 to 2004.

## Table A2

**Time-to-expiry and cash dividend and franking credit drop-off patterns in near futures contract mispricing**

| | Assuming cash and franking credits are worthless ($\gamma_1$=$\gamma_2$=0) | | Assuming cash and franking credits are partially valued ($\gamma_1$=0.78 and $\gamma_2$=0.72) | | Assuming cash and franking credits are fully valued ($\gamma_1$=$\gamma_2$=1) | |
|---|---|---|---|---|---|---|
| | $M^n_t$ | $\|t\|$ | $M^n_t$ | $\|t\|$ | $M^n_t$ | $\|t\|$ |
| **Panel A: Descriptive statistics** | | | | | | |
| Mean | -22.0 | | -0.8 | | 5.8 | |
| Median | -24.3 | | -0.5 | | 5.7 | |
| st. dev. | 12.7 | | 3.7 | | 5.1 | |
| *N* | 930 | | 930 | | 930 | |
| **Panel B: Time-to-expiry and cash and franking credit drop-off patterns** | | | | | | |
| $\alpha_0$ | -1.05 | 1.12 | -1.05 | 1.12 | -1.05 | 1.12 |
| $Cash^n_t$ | -0.78 | 18.32* | 0.00 | 0.00 | 0.22 | 5.19* |
| $Franking^n_t$ | -0.72 | 5.61* | -0.00 | 0.00 | 0.28 | 2.20* |
| $D_1$ | 0.87 | 0.93 | 0.87 | 0.93 | 0.87 | 0.93 |
| $D_2$ | 0.11 | 0.15 | 0.11 | 0.15 | 0.11 | 0.15 |
| $D_4$ | -0.14 | 0.21 | -0.14 | 0.21 | -0.14 | 0.21 |
| $D_5$ | 0.49 | 0.60 | 0.49 | 0.60 | 0.49 | 0.60 |
| $D_6$ | 0.43 | 0.44 | 0.43 | 0.44 | 0.43 | 0.44 |
| adj $R^2$ | 0.92 | | 0.00 | | 0.49 | |
| *F* | 1,511.30* | | 1.20 | | 127.91* | |

Panel A presents some summary statistics for the average intraday mispricing of the near futures contract ($M^n_t$). Panel B presents the results of the regression equation $M^n_t = \delta_0 + \gamma_1 Cash^n_t + \gamma_2 Franking^n_t + \sum \delta_i D_i + \varepsilon_t$ where $Cash^n_t$ is the theoretical value of cash dividends paid out by the stocks over the remaining life of the contract defined as the difference between the theoretical price of the futures contract estimated from zero dividends and the theoretical price of the futures contract estimated using cash dividends, $Franking^n_t$ is the theoretical value of franking credits paid out by the stocks over the remaining life of the contract defined as the difference between the theoretical price of the futures contract estimated from cash dividends and the theoretical price of the futures contract estimated using gross dividends and $D_i$ is a time-to-expiry dummy variable. *Denotes significance at the 5% level.

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